University of Notre Dame Calculus III

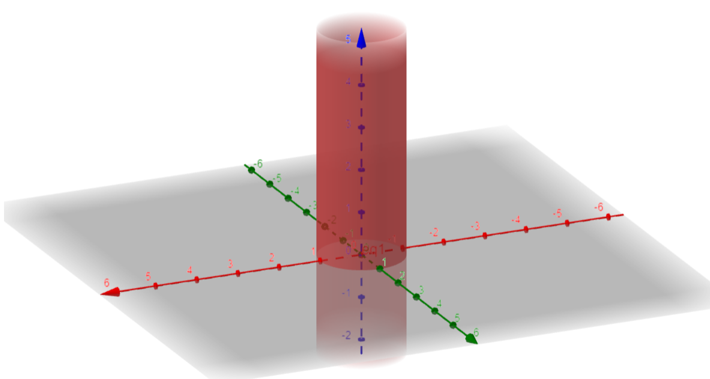
Cylinders and Quadric Surfaces

The point of covering this section* is to get you familiar with some of the surfaces we will be working with in the future. We will not cover the material as in-depth as the book does.

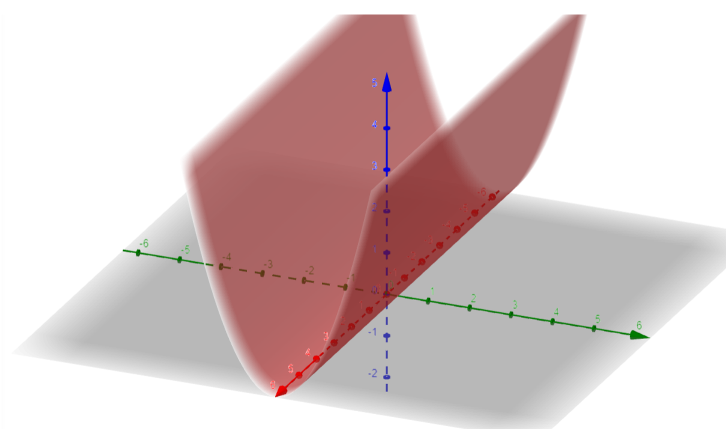
Cylinders

A cylinder is a surface consisting of all lines (called rulings) parallel to a given line and which pass through a given plane curve. In a lot less, fancy words, this essentially means you can roll up a piece of paper to look like the surface (well, a piece of it anyway).

Ex. a. $x^2 + y^2 = 1$



b. $z = y^2$



Quadric Surfaces

A quadric surface is simply the graph of a second degree polynomial in x, y, z. The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0\,.$$

However, through rotations and translations, they can all be made to look like

$$Ax^2 + By^2 + Cz^2 + J = 0$$

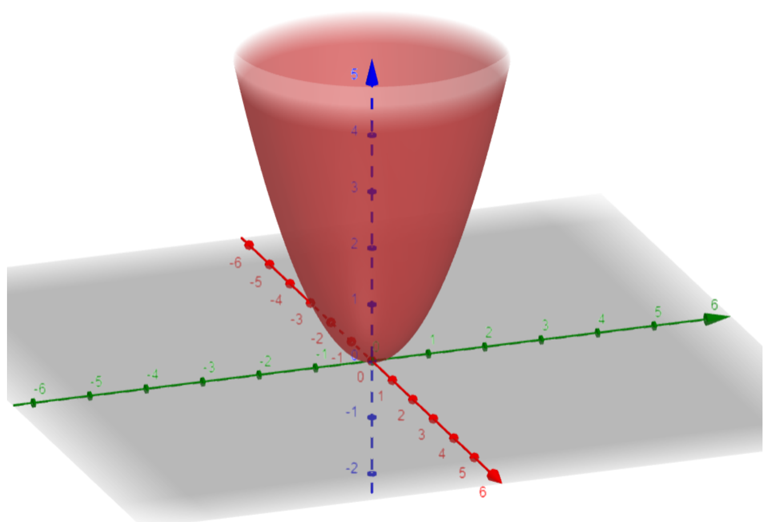
or

$$Ax^2 + By^2 + Iz = 0$$

Let's look at the ones of which will show up for us later:
Elliptic Paraboloids:

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

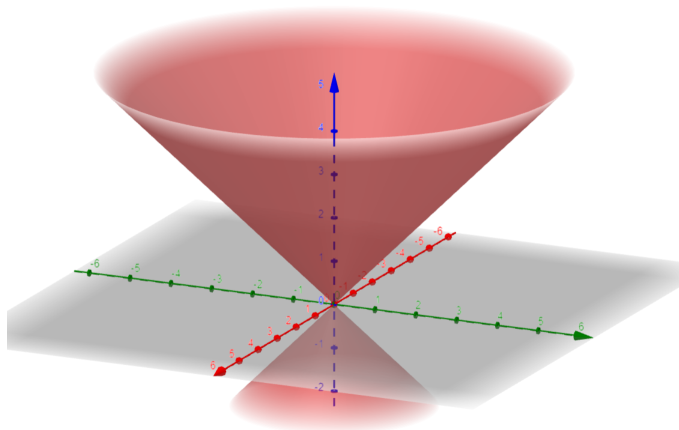
Ex: $z = 2x^2 + y^2$ These functions look similar to bowls.



Cones:

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Ex: $z^2 = x^2 + y^2$



Ellipsoids:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(Note: If $a = b = c$, this is a sphere)
Ex: $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$ These sorta look like rugby balls.