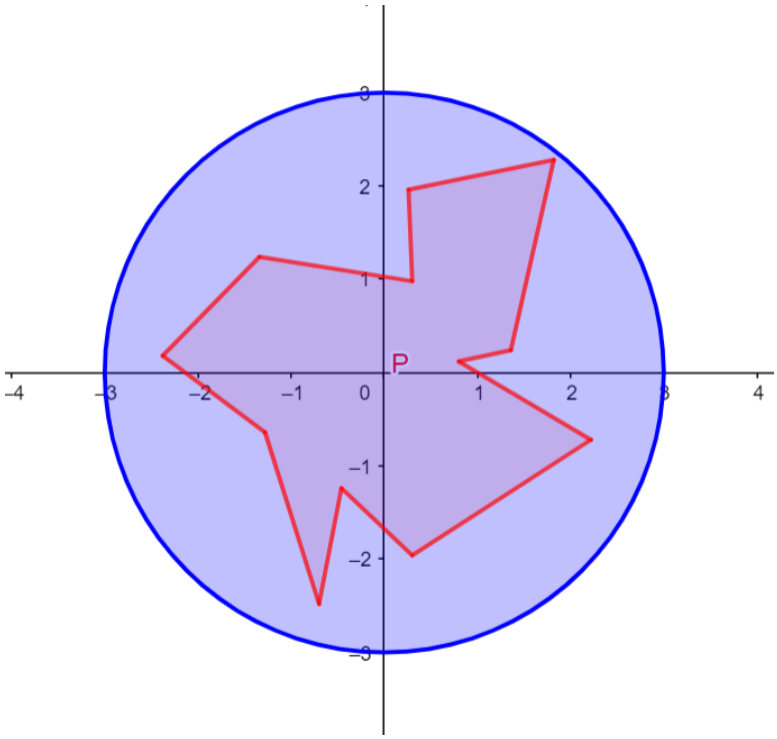
# University of Notre Dame Calculus III

LECTURE 15:ABSOLUTE MAXIMUMS AND MINIMUMS

We can also ask about extremum when we restrict the domain of $f$. First some definitions.

Closed sets are sets which contain all of their boundary points.

A set is bounded in $\mathbb{R}^2$ if it can fit in a disk.




The set of points bounded by polygon $P$ is bounded by the disc defined by all points $x^2 + y^2 \leq 9$.

An unbounded example is

$$\{(x, y) | x \geq a, b \leq y \leq c\}$$

**Procedure for finding extrema on closed and bounded sets**

This is the analog to finding extrema of $y = f(x)$ on $[a, b]$.
To find the absolute maximum and minimum values of a continuous function $f = f(x, y)$ on a closed and bounded set $D$:

1. Find the values of $f$ at critical points of $f$ inside $D$
2. Find the extreme values of $f$ on the boundary
3. The largest value from the first two steps is the absolute max and the smallest is the absolute min

**Example 1.** Find the absolute maximum and minimum values of $f(x, y) = x + y - xy$ on the closed and bounded set $D$ which is the closed triangle with vertices $(0,0)$, $(0,2)$, and $(4,0)$.

**Solution:**

**Example 2.** A cardboard box without a lid is to have a volume of $3200cm^3$. Find the dimensions of the box which uses the least amount of cardboard.

**Solution:**

**Extra Examples**

1. Does the function $f = x^2 - y^2$ have any local mininima or maxima?
2. Find the local min/max of $f = x^4 + y^4 - 4xy + 1$.
3. Find and classify all critical points of the function $f(x, y) = x^3 + y^3 - x - y$
4. Find the absolute maximum and absolute minimum values of

$$f(x, y) = xy - x - y$$

   on the triangular region with vertices $(0,0), (2,0), (2,4)$ and the point(s) where these extreme values are achieved.

5. Find the absolute maximum values of

$$f(x, y) = xy - x^2 + y$$

   inside the region bounded by $y = x^2$ and $y = 4$ and the point(s) where these extreme values are achieved.

6. Find the absolute maximum and absolute minimum values of

$$f(x, y) = x^2 + y^2 + y$$

   on the disk $x^2 + y^2 \leq 1$ and the point(s) where these extreme values are achieved.

7. Find the absolute maximum and absolute minimum values of

$$f(x, y) = (4 - y^2)e^{2x^2}$$

   over the region $4x^2 + y^2 \leq 4$ and the point(s) where these extreme values are achieved.