University of Notre Dame Calculus III

LECTURE 17: LAGRANGE MULTIPLIERS WITH 2 CONSTRAINTS

Two Constraints

Suppose we want to extremize $f(x,y,z)$ subject to two constraints: $g(x,y,z)=c$ and $h(x,y,z)=k$. Geometrically, we are extremizing f along the curve of intersection of $g=c$ and $h=k$. Now, we still have that ∇f is perpendicular to the curve of intersection at an extreme point, P, but it isn't necessarily perpendicular to both $g=c$ and $h=k$ at this point. However, since both ∇g and ∇h are perpendicular to the curve at this point, we find that $\nabla f(P)=\lambda\nabla g(P)+\mu\nabla h(P)$. To summarize, we now have to solve

$$\begin{cases} \nabla f(x,y,z)=\lambda\nabla g(x,y,z)+\mu\nabla h(x,y,z) \\ g(x,y,z)=c \\ h(x,y,z)=k \end{cases}$$

Example 1. Find the points on the conic section* determined by $z^2=x^2+y^2$ and $z=x+y+2$ which are closest to the origin.

Solution:

Extra Examples

1. Find the Lagrange multipliers one needs to solve in order to find the minimum of the function

$$f\left(x,y,z\right)=x^2+y^2+z^2$$

on the curve of intersection of the surfaces $y^2-z^2=1$ and $x-y=1$.

2. Find the absolute maximum and absolute minimum values of

$$f(x,y,z)=2x+y$$

with respect to the constraints $g(x,y,z)=2x^2+z^2=4$ and $h(x,y,z)=2x+y+3z=6$ and the point(s) where these extreme values are achieved.