University of Notre Dame Calculus III

LECTURE 21: TRIPLE INTEGRALS USING CYLINDERICAL COORDINATES

Cylindrical Coordinates

Example 1. Compute the volume of the solid bounded by $z = x^2 + y^2$ and $z = 4$.

Solution:

Notice how this integral was done in polar coordinates, but with z just tacked on. This is exactly what cylindrical coordinates are:

$$(r, \theta, z) \qquad \text{where} \qquad x = r\cos\theta, \;\; y = r\sin\theta, \;\; z = z$$

As z is just tacked on, we find

$$\boxed{dV = r\,dr\,d\theta\,dz}$$

Cylindrical coordinates can help with triple integrals as polar did with double, as the previous example shows.

Example 2.

a) Write the point with cyindrical coordinates $\left(4, \frac{\pi}{3}, -2\right)$ in cartesian coordinates

b) Write the point with cartesian coordinates $(-2, 2\sqrt{3}, 3)$ in cylindrical coordinates.

Solution:

Extra Problems

1. Describe the surface whose equation in cylindrical coordinates is $z = r$.
2. Let $D \subset \mathbb{R}^2$ be the region given in polar coordinates by
$$D = \left\{(r,\theta) \mid \alpha \le \theta \le \beta,\ h_1(\theta) \le r \le h_2(\theta)\right\},$$
and let $E \subset^3$ be the solid region given by
$$E = \{(x, y, z) \mid (x, y) \in D,\ u_1(x, y) \le z \le u_2(x, y)\}.$$
Write the integral $\iiint_E f(x, y, z)\,dV$ as a triple integral in cylindrical coordinates.
3. Evaluate $\iiint_E \sqrt{x^2 + y^2}\,dV$, where E is the region between the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.