University of Notre Dame Calculus III

LECTURE 32: FLUX INTEGRAL

Flux

The concept of flux is measuring the rate at which something flows through a surface (i.e. air through a butterfly net).

$$d\vec{S} = \vec{n}\,dS = \frac{\vec{r}_u \times \vec{r}_v}{||\vec{r}_u \times \vec{r}_v||}(||\vec{r}_u \times \vec{r}_v\,dA) = (\vec{r}_u \times \vec{r}_v)\,dA$$

Definition 1. *If $\vec{F}$ is a continuous vector field defined on a surface S which has orientation $\vec{n}$, then the flux of $\vec{F}$ across S is*

$$\int\int_S \vec{F}\cdot d\vec{S} = \int\int_S \left(\vec{F}\cdot\vec{n}\right)\,dS = \int\int_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r}_v)\,dA$$

Where S is parametrized by $\vec{r}(u,v)$ and $\vec{n} = \frac{\vec{r}_u\times\vec{r}_v}{||\vec{r}_u\times\vec{r}_v||}$.

Example 1. Find the flux of $\vec{F} = \langle x, y, z\rangle$ across the helicoid parametrized by $\vec{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$, $0 \le u \le 1$, $0 \le v \le \frac{\pi}{2}$ with upward orientation.

Solution:

Example 2. Compute $\int\int_S \vec{F}\cdot d\vec{S}$ where $\vec{F} = \vec{G}$. $\vec{G} = \langle -2yz, y, 3x\rangle$, and S is the piece of the paraboloid $z = 5 - x^2 - y^2$ above the plane $z = 1$ with the upward orientation.

Solution: